This examination paper consists of THREE sections: Module 1.1, Module 1.2 and Module 1.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. DO NOT open this examination paper until instructed to do so.
2. Answer ALL questions from the THREE sections.
3. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper
SECTION A (MODULE 1.1)

Answer BOTH questions.

1. (a) Given that both \((x - 1)\) and \((x - 2)\) are factors of \(f(x) = x^3 + mx + n\), find the constants \(m\) and \(n\), and the third factor of \(f(x)\). [10 marks]

(b) Find the constants \(p\), \(q\) and \(r\) such that

\[2y^2 - 9y + 14 = p(y-1)(y-2) + q(y-1) + r.\] [8 marks]

(c) (i) Find the range of values of \(x \in \mathbb{R}\) for which

\[|2x - 3| \leq 5.\]

Hence, determine

(ii) the LEAST possible value of \(x + 1\) [1 mark]

(iii) the GREATEST possible value of \(x + 1\). [1 mark]

Total 25 marks

2. (a) (i) Express \(f(x) = 12x - 2x^2\) in the form \(A + B(x+p)^2\) where \(A\), \(B\) and \(p\) are real numbers, and find the maximum value of \(12x - 2x^2\). [7 marks]

(ii) Hence, sketch the graph of \(f(x) = 12x - 2x^2\), showing clearly its main features. [5 marks]

(iii) Show that \(f(x) = 12x - 2x^2\) is NOT one-to-one. [2 marks]

(b) (i) a) Copy and complete the following table for the function \(f(x) = \sin x, 0 \leq x \leq 2\pi\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>(\frac{\pi}{2})</th>
<th>(\pi)</th>
<th>(\frac{3\pi}{2})</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Sketch the graph of \(f\). [4 marks]

(ii) On a separate diagram, sketch the graph of \(f(x) = |\sin x|, 0 \leq x \leq 2\pi\). [4 marks]

(iii) By comparing the diagrams in (b)(i) and (ii) above, determine the solution set of the equation \(\sin x = |\sin x|, 0 \leq x \leq 2\pi\). [3 marks]

Total 25 marks

GO ON TO THE NEXT PAGE
3. (a) In the diagram below (not drawn to scale), $PQ$ is perpendicular to $AQB$.

Find

(i) the equation of the line $AB$ [4 marks]

(ii) the equation of the line $PQ$ [4 marks]

(iii) the coordinates of the point $Q$. [4 marks]

(b) Solve, for $0^\circ \leq \theta \leq 180^\circ$, the equation

$$6 \cos^2 \theta + \sin \theta = 4.$$ [7 marks]

(c) Solve, for $0 \leq x \leq \pi$, the equation

$$\sin x + \sin 3x = 0.$$ [6 marks]

Total 25 marks
4. (a) A complex number, \(z\), is expressed in the form \(x + iy\), where \(x, y \in \mathbb{R}\). Express the complex number, \(w = \frac{z-1}{z+2}\), in a similar form. [8 marks]

(b) The argument of \(w\) is \(\frac{\pi}{4}\).

(i) Find the equation connecting \(x\) and \(y\) in the form
\[ax^2 + by^2 + cx + dy + f = 0\]
where \(a, b, c, d, f\) are integers. [4 marks]

(ii) Show that the equation in (i) represents a circle, \(C\). [3 marks]

(iii) Determine the centre and radius of the circle \(C\). [4 marks]

(c) The diagram below (not drawn to scale) shows a parallelogram \(OLMN\) whose diagonals, \(OM\) and \(LN\), intersect at \(P\). The position vectors of \(L\) and \(N\) relative to the origin, \(O\), are \(-3i + 6j\) and \(2i + 3j\) respectively.

Find the position vector of \(P\). [6 marks]

Total 25 marks
SECTION C (MODULE 1.3)

Answer BOTH questions.

5. (a) Evaluate \( \lim_{{x \to 3}} \frac{x^2 - 2x - 3}{x^2 - 4x + 3} \). [4 marks]

(b) Determine the values of \( x \in \mathbb{R} \) for which the function \( \frac{x + 2}{x(x + 1)} \) is NOT continuous. [3 marks]

(c) Given that \( y = \frac{x^2 - 1}{x^2 + 1} \),

(i) find \( \frac{dy}{dx} \) in terms of \( x \) [5 marks]

(ii) show that \( x(x^2 + 1) \frac{dy}{dx} - 4y = \frac{4}{x^2 + 1} \). [5 marks]

(d) By investigating the sign of \( f''(x) \), determine the range of real values of \( x \) for which \( x^3 - 5x + 3 \) is decreasing [8 marks]

Total 25 marks

6. (a) Find the stationary point(s) of the curve, \( f(x) = x^3 - 3x + 2 \). [6 marks]

(b) Determine the nature of the stationary point(s). [3 marks]

(c) Show that the curve \( f(x) \) touches the \( x \)-axis at \( x = 1 \). [4 marks]

(d) Sketch the curve, \( f(x) = x^3 - 3x + 2 \), \( -2 \leq x \leq 2 \). [6 marks]

(e) Find the area bounded by this curve and the \( x \)-axis for \( -2 \leq x \leq 1 \). [6 marks]

Total 25 marks

END OF TEST